

Assignment 1

Due: 11:59pm, September 16, 2025

Please read the **Grading policies for assignments** on the course web site.

1. **Distinguishing between pairs of states [15 points; 5 for each part].** In each case, one of the two given states is prepared and sent to you (you are not told which one). Your goal is to guess which of the two states it is. You are allowed to perform any measurement operation on the state to help you with this goal.

Describe your distinguishing procedure in terms of a unitary operation followed by a measurement (in the computational basis) and give its worst-case success probability. (Your assigned grade will depend on how close your success probability is to optimal.)

(a) $|0\rangle$ vs. $\frac{1}{2}|0\rangle + \frac{\sqrt{3}}{2}|1\rangle$

(b) $\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$ vs. $\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}i|1\rangle$ (where $i = \sqrt{-1} = e^{i\pi/2}$)

(c) $\frac{1}{2}|00\rangle + \frac{1}{2}|01\rangle + \frac{1}{2}|10\rangle - \frac{1}{2}|11\rangle$ vs. $\frac{1}{2}|00\rangle - \frac{1}{2}|01\rangle - \frac{1}{2}|10\rangle - \frac{1}{2}|11\rangle$

2. **Simple operations on quantum states [15 points; 5 each].**

(a) If H is applied to both qubits of state $\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$, what is the resulting state?

(b) If H is applied to the *first two* qubits of state $\frac{1}{2}|000\rangle - \frac{1}{2}|011\rangle - \frac{1}{2}|101\rangle - \frac{1}{2}|110\rangle$, what is the resulting state?

(c) Find a 2×2 unitary operation U with the property that

$$U \otimes U \left(\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle \right) = \frac{1}{\sqrt{2}}|01\rangle + \frac{1}{\sqrt{2}}|10\rangle.$$

In words, applying U to both qubits of state $\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$ should result in the state $\frac{1}{\sqrt{2}}|01\rangle + \frac{1}{\sqrt{2}}|10\rangle$.

3. **Entangled states and product states [12 points; 4 each].** For each two-qubit state below, either express it as a product of two one-qubit states (i.e., state vectors) or show that such a factorization is impossible (in the latter case, the qubits are *entangled*).

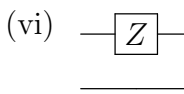
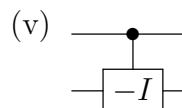
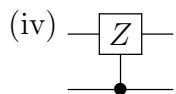
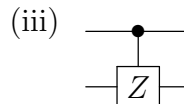
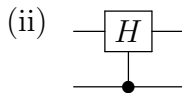
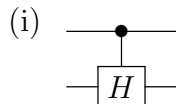
(a) $\frac{60}{61}|00\rangle + \frac{11}{61}|11\rangle$

(b) $\frac{1}{2}|00\rangle + \frac{1}{2}i|01\rangle - \frac{1}{2}i|10\rangle + \frac{1}{2}|11\rangle$

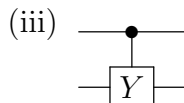
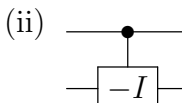
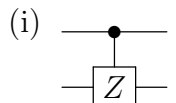
(c) $\frac{1}{2}|00\rangle + \frac{1}{2}|01\rangle + \frac{1}{2}|10\rangle - \frac{1}{2}|11\rangle$

4. **Simple quantum circuits with one gate [18 points].**

- (a) [6 points; 1 each] In each of the six cases below, give the 4×4 unitary matrix corresponding to the circuit:



- (b) [12 points; 4 each] Suppose that we start with the product state $|+\rangle|0\rangle$, and apply a controlled- X (a.k.a. CNOT) gate to it. The resulting state is $\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$, which is of course entangled. Call a 2-qubit gate *entangling* if there exists at least one product state that the gate maps to an entangled state. In each of the three cases below, state whether or not the gate is entangling and justify your answer.



5. **(This is an optional question for bonus credit)**

A state distinguishing problem involving three states [8 points].

Consider the state distinguishing problem where you are given one of these three states

$$|\phi_0\rangle = |0\rangle$$

$$|\phi_1\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

$$|\phi_2\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$$

and your goal is to determine which state you received. Give a procedure whose worst-case success probability is $\frac{64}{100}$ (i.e., whose success probability is $\geq \frac{64}{100}$ in all three cases).

There is a solution that can be explained within one page. If you submit a solution to this question then please do not exceed two pages.

Please see the comments about bonus questions in the *Grading policies for assignments*.