Assignment 4
Due: 11:59pm, October 19, 2021

1. **Function $f : \{00, 01, 10\} \to \{0, 1\}$ with an even number of 1s [15 points].** Suppose that you are given a black-box that computes a function $f : \{00, 01, 10\} \to \{0, 1\}$ such that the number of inputs where the function takes the value 1 is *even*. The function takes two bits as input and if it is queried at the "out of range" point 11 then it takes the value 0. In summary, there are four possible functions with this property:

x	$f_{000}(x)$
00	0
01	0
10	0
11	0

x	$f_{110}(x)$
00	1
01	1
10	0
11	0

x	$f_{101}(x)$
00	1
01	0
10	1
11	0

x	$f_{011}(x)$
00	0
01	1
10	1
11	0

Your goal is to determine which of the four functions your black-box is, where you are allowed to query f at any point in $\{00, 01, 10, 11\}$.

   (a) How many classical queries are there needed for this problem? Justify your answer.

   (b) Give a quantum algorithm that solves this problem making one query to the function and explain why it works.

2. **Question related to a detail of Simon's algorithm [15 points].** Suppose that $r, s \in \{0, 1\}^n$ are both non-zero and $r \neq s$. Suppose that $a \in \{0, 1\}^n$ is arbitrary and you are given one copy of the n-qubit state

$$\tfrac{1}{2}|a\rangle + \tfrac{1}{2}|a \oplus r\rangle + \tfrac{1}{2}|a \oplus s\rangle + \tfrac{1}{2}|a \oplus r \oplus s\rangle. \tag{1}$$

What is the probability distribution on $\{0, 1\}^n$ that describes the result of applying $H^{\otimes n}$ to this state and then measuring in the computational basis? Please show your calculations.

3. **(This is an optional question for bonus credit)**
Function $f : \{00, 01, 10\} \to \{0, 1\}$ with an even number of 1s revisited [8 points]. As in question 1, you are given a black-box computing a function $f : \{00, 01, 10\} \to \{0, 1\}$ such that the number of inputs where the function has value 1 is even. However, now the value at the "out of range" point 11 is *arbitrary*. Now there are these four possibilities

x	$f_{000}(x)$
00	0
01	0
10	0
11	$*$

x	$f_{110}(x)$
00	1
01	1
10	0
11	$*$

x	$f_{101}(x)$
00	1
01	0
10	1
11	$*$

x	$f_{011}(x)$
00	0
01	1
10	1
11	$*$

where $*$ means that the bit could be *either* 0 *or* 1 *and you don't know which it is.* Is there a quantum algorithm that solves *this* version of the problem (determining which of the four cases) with one single query? If your answer is yes then you must give the quantum algorithm. If your answer is no then you must prove that there is no one-query quantum algorithm.