Assignment 1
Due date: 11:59pm, September 21, 2021

1. **Distinguishing between pairs of states [16 points; 4 for each part].** In each case, one of the two given states is prepared and sent to you (you are not told which one; each case arises with probability $\frac{1}{2}$). Your goal is to guess which of the two states it is. You are allowed to perform any measurement operation on the state to help you with this goal.

   Describe your distinguishing procedure as a unitary operation followed by a measurement (in the computational basis) and give its average-case success probability. (Your assigned grade will depend on how close your average-case success probability is to optimal.)

   (a) $|0\rangle$ vs. $-\frac{1}{2}|0\rangle + \frac{\sqrt{3}}{2}|1\rangle$

   (b) $\frac{1}{\sqrt{2}}|0\rangle + \frac{i}{\sqrt{2}}|1\rangle$ vs. $\frac{1}{\sqrt{2}}|0\rangle + \frac{-i}{\sqrt{2}}|1\rangle$ (where $i = \sqrt{-1} = e^{i\pi/2}$)

   (c) $\frac{1}{\sqrt{2}}|0\rangle + \frac{\omega}{\sqrt{2}}|1\rangle$ vs. $\frac{1}{\sqrt{2}}|0\rangle + \frac{-\omega}{\sqrt{2}}|1\rangle$ (where $\omega = e^{i\pi/4}$)

   (d) $|00\rangle$ vs. $\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$ (in this case you're given two qubits).

2. **Applying 1-qubit gates to 2-qubit states [14 points; 7 each].** Consider these two 2-qubit states:

$$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$
$$\frac{1}{\sqrt{2}}|00\rangle - \frac{1}{\sqrt{2}}|11\rangle$$

   (a) For each of the two states, calculate the state that results if a Hadamard gate H is applied to the *second* qubit (i.e., $I \otimes H$). Recall that $H = \frac{1}{\sqrt{2}}\left[\begin{smallmatrix} 1 & 1 \\ 1 & -1 \end{smallmatrix}\right]$.

   (b) For each of the two states, calculate the state that results if a Hadamard gate H is applied to *both* of the two qubits (i.e., $H \otimes H$).

3. **(This is an optional question for bonus credit)**
   Classical *bit* strategies for communicating a trit [8 points].
   Towards the end of video lecture 1 (and in the lecture notes, Primer, section 5), we consider the problem where Alice receives a trit $a \in \{0, 1, 2\}$ and the goal is to communicate this trit to Bob. Here we consider the case where Alice is allowed to send (only) one **classical bit** to Bob. Assume that both Alice and Bob can make random decisions in their strategies; however, assume that they have separate random sources so their randomness is *uncorrelated*.

   In the video/notes, we saw a strategy whose worst-case success is probability is $\frac{1}{2}$. What's the highest possible worst-case success probability achievable by a classical bit strategy? Any answer must be justified by a clearly explained proof.