Assignment 8

Due date: **Tuesday** 11:59pm, November 17, 2020

1. **Kraus operators for the reset channel [8 points].** Consider the quantum channel that takes a qubit as input and produces as output a qubit in state $|0\rangle\langle 0|$ (regardless of what the input state is). In Lecture 15, we saw a description of this channel in the Stinespring form. Give a description of this channel in the Kraus form. That is, give Kraus operators for this channel.

2. **Distinguishing between $|0\rangle$ vs. $|+\rangle$ revisited [10].** We have previously considered the guess-the-state game, with $|0\rangle$ and $|+\rangle$. A qubit is set to one of these two states (each selected with probability 1/2) and then this qubit is provided to you. We already known that there is a measurement procedure for this that succeeds with probability $\cos^2(\pi/8) = (1 + \frac{1}{\sqrt{2}})/2 \approx 0.85$. Prove that there does not exist a POVM measurement that attains a higher success probability.

3. **A four-state distinguishing problem [12 points; 6 each].** Consider these four qutrit states:

$$|\psi_0\rangle = \frac{1}{\sqrt{3}}|0\rangle + \frac{1}{\sqrt{3}}|1\rangle + \frac{1}{\sqrt{3}}|2\rangle \quad (1)$$
$$|\psi_1\rangle = \frac{1}{\sqrt{3}}|0\rangle - \frac{1}{\sqrt{3}}|1\rangle - \frac{1}{\sqrt{3}}|2\rangle \quad (2)$$
$$|\psi_2\rangle = -\frac{1}{\sqrt{3}}|0\rangle + \frac{1}{\sqrt{3}}|1\rangle - \frac{1}{\sqrt{3}}|2\rangle \quad (3)$$
$$|\psi_3\rangle = -\frac{1}{\sqrt{3}}|0\rangle - \frac{1}{\sqrt{3}}|1\rangle + \frac{1}{\sqrt{3}}|2\rangle. \quad (4)$$

   Note that the inner product between any pair of these states is $-\frac{1}{3}$ and that these vectors are the vertices of a regular tetrahedron in $\mathbb{R}^3$. Consider the guess-the-state-state game, where a qutrit is set to one of these states (selected at random, each with probability 1/4) and the qutrit is provided to you. Your task is to perform a measurement procedure on the qutrit and then guess which state you received. These states are not orthogonal, so your procedure will be probabilistic.

   (a) Give a measurement in the Kraus form for this problem with as high a success probability as you can.

   (b) Give a measurement in the Stinespring form for this problem with as high a success probability as you can.

   (Note that, although we have seen a systematic way of analyzing the guess-the-state game for two states, we have seen no such methodology for more than two states. You are not required to prove optimality. But this also means that you might might not know that your procedure is the best possible.)

4. **(This is an optional question for bonus credit)**
   Optimality of measurement procedure in question 3 [8 points]. Prove that your measurement procedure for question 3 is optimal.