Assignment 2
Due date: 11:59pm, September 24, 2020

1. **Simple operations on quantum states [12 points; 3 for each part].** Let $\theta \in [0, 2\pi]$ and R_θ be the 2×2 rotation matrix

$$R_\theta = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}. \tag{1}$$

In each case, describe the resulting state after the operation is performed:

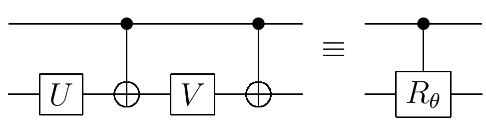
   (a) Apply R_θ to the qubit in state $\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$.

   (b) Apply R_θ to the *first* qubit of state $\frac{1}{\sqrt{2}}|00\rangle - \frac{1}{\sqrt{2}}|11\rangle$.

   (c) Apply R_θ to *both* qubits of state $\frac{1}{\sqrt{2}}|00\rangle - \frac{1}{\sqrt{2}}|11\rangle$.

   (d) Apply $\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix}$ to *both* qubits of state $\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$ (where $i = \sqrt{-1}$)

2. **Simulating a controlled-rotation with CNOT and one-qubit gates [8 points].** Let $\theta \in [0, 2\pi]$ and R_θ be the rotation matrix defined in question 1. Suppose that we want to simulate a controlled-R_θ gate using only the CNOT gate and 1-qubit gates. Consider a construction of this form:



Show that there exist 1-qubit unitary operations U and V such that the circuit simulates the controlled-R_θ gate. (Hint: consider setting U and V to rotation matrices with carefully chosen angles.)

3. **Circuit for constructing a state [10 points].** Give a circuit consisting of one CNOT gate and two 1-qubit gates that transforms the state $|00\rangle$ to $\frac{1}{2}|00\rangle + \frac{1}{2}|01\rangle + \frac{1}{2}|10\rangle - \frac{1}{2}|11\rangle$.

4. **(This is an optional question for bonus credit)**
Qubit strategies for communicating a trit [8 points].
In Lecture 1, slides 11–13, we considered the problem where Alice receives a trit $a \in \{0, 1, 2\}$ and the goal is to communicate this trit to Bob. Here we consider the case where Alice is allowed to send (only) **one qubit** to Bob. What's the highest possible worst-case success probability achievable by a qubit strategy? Any answer must be justified. (You may assume that the maximum average-case success probability is $\frac{2}{3}$, as was stated in the lecture.)