Assignment 1
Due date: 11:59pm, September 17, 2020

1. **Distinguishing between pairs of qubit states [15 points; 5 for each part].** In each case, one of the two given states is randomly selected (probability 1/2 each) and given to you. You are not told which one it is. Your goal is to guess which state was selected with as high a probability as you can achieve. Describe your distinguishing procedure as a unitary operation followed by a measurement (in the computational basis) and give its success probability. (Your assigned grade will depend on how close your distinguishing procedure is to optimal.)

   (a) $|0\rangle$ and $|+\rangle$ (recall that $|+\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$).

   (b) $|0\rangle$ and $-\frac{1}{2}|0\rangle + \frac{\sqrt{3}}{2}|1\rangle$.

   (c) $\frac{1}{\sqrt{2}}|0\rangle + \frac{i}{\sqrt{2}}|1\rangle$ and $\frac{1}{\sqrt{2}}|0\rangle + \frac{-i}{\sqrt{2}}|1\rangle$ (where $i = \sqrt{-1}$).

2. **Product states versus entangled states [15 points; 5 each].** In each case, either express the 2-qubit state as a tensor product of two 1-qubit states or prove that it cannot be expressed this way:

   (a) $\frac{1}{2}|00\rangle + \frac{1}{2}|01\rangle + \frac{1}{2}|10\rangle - \frac{1}{2}|11\rangle$

   (b) $\frac{1}{2}|00\rangle + \frac{1}{2}|01\rangle - \frac{1}{2}|10\rangle - \frac{1}{2}|11\rangle$

   (c) $\frac{1}{\sqrt{3}}|00\rangle + \frac{1}{\sqrt{3}}|01\rangle + \frac{1}{\sqrt{3}}|10\rangle$

3. **(This is an optional question for bonus credit)**
   Classical bit strategies for communicating a trit [8 points].
   In Lecture 1, slides 11–13, we consider the problem where Alice receives a trit $a \in \{0, 1, 2\}$ and the goal is to communicate this trit to Bob. Here we consider the case where Alice is allowed to send (only) one **classical bit** to Bob. Both Alice and Bob can make random decisions in their strategies; however, we assume that they have separate random sources so their randomness is *uncorrelated*. In the lecture, we saw a strategy whose worst-case success is probability is 1/2. What's the highest possible worst-case success probability achievable by a classical bit strategy? Any answer must be justified.

   (In the next assignment, we will continue this line, with the corresponding question about qubit strategies.)